Error Models of Bayesian Prediction using Numerical Simulation

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Abstract: Numerical simulations are widely used to predict future events. Uncertainty quantification of numerical simulation predictions can be assessed using a Bayesian framework proposed in our previous works [1, 2] and by several other authors [3, 4]. Under this framework, one first reduces the uncertainties on the unknown model parameters through an inverse problem and assigns posterior probability to model parameters. The posterior probability is obtained by updating the prior with the historical observation. Then one conducts forward simulation to predict. Although the Bayesian formulation is very simple, its success depends largely on robust and reliable error models from which the likelihoods of observations are derived to feed the Bayes formula in the inverse problem. The error of a numerical model comprises solution errors and modeling errors. The solution errors are the result of a finite accuracy approximation to the governing equations describing continuum phenomenon. The modeling errors could be due to approximations in the equations and the physics they represent. Assessment of the modeling errors, often referred to as model validation, is very important but is not discussed here. The focus of this article and some of our previous works is on the solution errors.

Solution error could be a dominating force of the entire numerical simulation error in applications in which only coarse grid solutions are afforded. One of such applications is prediction of future production of oil reservoirs, in which our Bayesian framework requires forward simulations to be done over a large ensemble of different reservoir geologies. Other applications include shock wave physics. Solution errors could be assessed by comparing the coarse grid solution and more accurate solutions. A probability model of solution errors can be established with a number of observations. Two issues on constructing such probability models will be discussed in this paper. First, the choice of dimensionality of the probability model, an over simplified model misses the important features of the error structure and an over complex model is less robust and requires large sample size. Second, the transferability of error models are discusses through a study on shock wave physics. We will illustrate how error models of complex problems may be constructed from error models of more basic components.

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